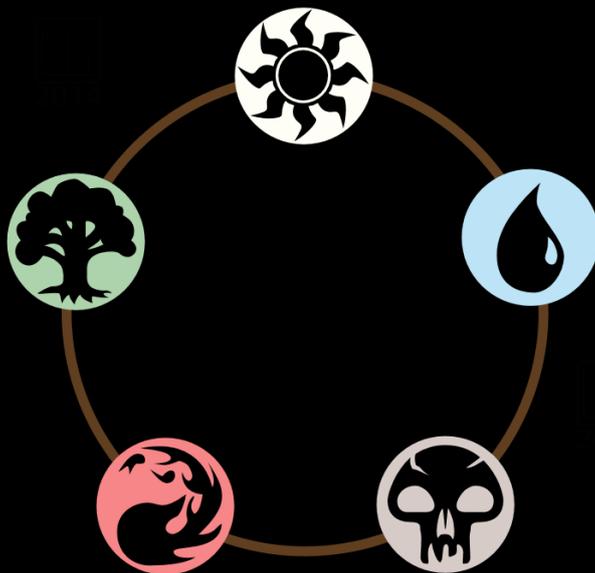


# MAGIC The Gathering Old School



## How Many Lands Do You Need to Consistently Hit Your Land Drops?

By  
*Frank Karsten*

BY **FRANK KARSTEN** / MAY 30, 2017

I have seen decks with as few as 1 land and decks with as many as many as 42. But how many lands does a 60-card deck actually need? In this article, I will run the numbers.

I will use two different methodologies. The first approach is based on raw probability calculations. For example, I will determine the probability of hitting 4 lands by turn 4 under a certain mulligan strategy. The second method is a linear regression between the number of lands and the average converted mana costs in recent top-performing decks. I'll conclude with a table that summarizes the main findings and recommendations. I sprinkled in some random thoughts and observations throughout. Let's get to it!

## Method 1: The Mulligan Rule

In this article, I will use phrases like “hit your 4th land drop” to mean that you played lands on turns 1, 2, 3, and 4. To determine the probability of hitting your 4th land drop, I will assume the following mulligan strategy:

- You mulligan any 7-card hand with 0, 1, 6, or 7 lands.
- You mulligan any 6-card hand with 0, 1, 5, or 6 lands.
- You mulligan any 5-card hand with 0 or 5 lands.
- You keep any 4-card hand.
- After a mulligan, you always scry a land to the top and a spell to the bottom.

In my experience, this strategy is reasonable for a wide range of decks. There are some low-curve decks that might want to keep a 1-lander and/or mulligan a 5-lander, and there are some high-curve decks that might want to mulligan a 2-lander and/or keep a 6-lander. You may want to keep that in mind when interpreting the eventual results. But I decided to keep things simple by enforcing the same mulligan strategy for every type of deck—this makes it easier to compare the numbers for different land counts.

## The Formula

Given a 60-card deck with a given number of lands, I started by determining the probability of keeping any opening hand with a certain number of cards and a certain number of lands. These are basic hypergeometric probabilities, multiplied (for hands of 6 cards or fewer) by the probability of taking a mulligan down to that many cards under the above-described mulligan strategy. Naturally, the resulting probabilities over all opening hand sizes and all land counts sum up to one.

Then, for any opening hand I might keep, I asked, “what is the probability to find at least a certain number of lands after a certain number of draw steps?” For instance, finding at least 5 lands after 4 draw steps would represent the probability of hitting your first 5 land drops on the play. This calculation is also fairly straightforward, but I need to be a little more careful in distinguishing all the cases.

Consider a 60-card library with  $L_{lb}$  lands. Consider an opening hand with  $N_{op}$  cards and  $L_{op}$  lands. The probability  $\mathbb{P}(\text{hit})$  of hitting at least  $L_{rq}$  lands after  $N_{dr}$  draw steps is given by

$$\mathbb{P}(\text{hit}) = \begin{cases} \sum_{\ell=L_{rq}-L_{op}}^{L_{op}+N_{dr}} H(\ell, N_{dr}, 60 - N_{op}, L_{lb} - L_{op}) & \text{if } L_{rq} > L_{op} \text{ and } N_{op} = 7; \\ \frac{L_{lb}-L_{op}}{60-N_{op}} \sum_{\ell=L_{rq}-1-L_{op}}^{L_{op}+N_{dr}-1} H(\ell, N_{dr}-1, 60 - N_{op} - 1, L_{lb} - L_{op} - 1) \\ + \left(1 - \frac{L_{lb}-L_{op}}{60-N_{op}}\right) \sum_{\ell=L_{rq}-L_{op}}^{L_{op}+N_{dr}} H(\ell, N_{dr}, 60 - N_{op} - 1, L_{lb} - L_{op}) & \text{if } L_{rq} > L_{op} \text{ and } N_{op} < 7; \\ 1 & \text{if } L_{rq} \leq L_{op}. \end{cases}$$

where  $H(a, b, c, d)$  represents the hypergeometric probability of hitting at least  $a$  successes in a sample of size  $b$  from a population of size  $c$  containing  $d$  successes.

Note that case involving mulligans distinguish between having a land on top, which happens with probability  $(L_{lb} - L_{op})/(60 - N_{op})$ , and having a spell on top, which happens with the complementary probability.

## The Results

<b>Lands in deck</b>	<b>P(2 lands by turn 2)</b>	<b>P(3 lands by turn 3)</b>	<b>P(4 lands by turn 4)</b>	<b>P(5 lands by turn 5)</b>	<b>Exp. opening hand</b>	<b>P(mana flood)</b>
<b>17</b>	96.8% / 95.6%	77.2% / 69.1%	51.1% / 41.0%	28.8% / 20.8%	6.458 cards	1.2%
<b>18</b>	97.7% / 96.8%	80.7% / 73.0%	56.3% / 45.9%	33.8% / 24.9%	6.530 cards	1.9%
<b>19</b>	98.4% / 97.7%	83.8% / 76.5%	61.2% / 50.6%	39.0% / 29.2%	6.594 cards	2.8%
<b>20</b>	98.9% / 98.3%	86.4% / 79.6%	65.8% / 55.2%	44.1% / 33.7%	6.649 cards	3.9%
<b>21</b>	99.2% / 98.8%	88.6% / 82.3%	70.0% / 59.6%	49.3% / 38.3%	6.697 cards	5.4%
<b>22</b>	99.5% / 99.2%	90.5% / 84.7%	73.9% / 63.7%	54.3% / 43.0%	6.738 cards	7.3%
<b>23</b>	99.6% / 99.4%	92.1% / 86.8%	77.4% / 67.7%	59.1% / 47.6%	6.772 cards	9.5%
<b>24</b>	99.8% / 99.6%	93.5% / 88.7%	80.6% / 71.3%	63.8% / 52.2%	6.801 cards	12.2%
<b>25</b>	99.8% / 99.7%	94.6% / 90.4%	83.5% / 74.7%	68.2% / 56.7%	6.825 cards	15.2%
<b>26</b>	99.9% / 99.8%	95.6% / 91.8%	86.0% / 77.9%	72.3% / 61.1%	6.844 cards	18.7%
<b>27</b>	99.9% / 99.8%	96.4% / 93.1%	88.3% / 80.8%	76.0% / 65.3%	6.858 cards	22.6%
<b>28</b>	99.9% / 99.9%	97.1% / 94.2%	90.2% / 83.4%	79.5% / 69.2%	6.868 cards	26.8%

***The first couple of columns contain two percentages:  
The first refers to the probability when you are on the draw, and the second refers to the probability when you are on the play.***

The “exp. opening hand” column indicates the expected size of your opening hand after all mulligans are said and done. The last column, “P(mana flood)” represents the probability of having drawn at least 8 lands by turn 7 when you are on the draw. This is only one way to represent mana flood, but it gives at least some indication.

Consider a deck that needs at least 3 lands to function that would like to play [Gideon, Ally of Zendikar](#) on turn 4, and that contains several copies of [Archangel Avacyn](#). For such a deck, I'd say that 25 lands is probably the right number. According to the table, you will almost always (94.6% on the draw, 90.4% on the play) hit 3 lands by turn 3 under the prescribed mulligan rule, which means that you can almost always play a reasonable game of Magic. The probability of hitting 4 lands by turn 4 is sufficient: 83.5% on the draw, 74.7% on the play. That's still not super consistent, but I wouldn't be comfortable going lower, and you can't reach the 90% level of consistency that I liked for [colored sources of mana](#) without going up to 28 lands.

## How about Limited?

All right, I can do that too.

Lands in deck	P(2 lands by turn 2)	P(3 lands by turn 3)	P(4 lands by turn 4)	P(5 lands by turn 5)	Exp. opening hand	P(mana flood)
12	97.9% / 97.0%	80.9% / 72.9%	55.5% / 44.7%	31.9% / 23.0%	6.547 cards	1.0%
13	98.8% / 98.2%	85.5% / 78.2%	63.2% / 52.2%	40.1% / 29.8%	6.639 cards	2.1%
14	99.3% / 99.0%	89.0% / 82.6%	70.1% / 59.3%	48.4% / 37.0%	6.714 cards	3.9%
15	99.6% / 99.4%	91.8% / 86.2%	76.2% / 65.8%	56.4% / 44.4%	6.772 cards	6.5%
16	99.8% / 99.7%	94.0% / 89.2%	81.3% / 71.7%	64.0% / 51.8%	6.817 cards	10.2%
17	99.9% / 99.8%	95.6% / 91.6%	85.6% / 77.0%	70.9% / 59.0%	6.851 cards	14.9%
18	99.9% / 99.9%	96.9% / 93.6%	89.2% / 81.6%	77.1% / 65.8%	6.873 cards	20.8%
19	100% / 99.9%	97.8% / 95.2%	92.1% / 85.5%	82.3% / 72.0%	6.887 cards	27.6%
20	100% / 99.9%	98.5% / 96.4%	94.3% / 88.9%	86.8% / 77.6%	6.891 cards	35.3%

The numbers for 17 lands in Limited are similar to the ones for 25 lands in Standard. This also makes sense because  $25 * 40/60 = 17.5$ . Since typical Limited decks need their first 3 land drops, would like to hit their 4th land drop, but don't necessarily need 5 lands by turn 5, the age-old standard of 17 lands indeed seems like a good number.

For Commander decks, the equivalent to 25 lands in a 60-card deck is  $25 * 99/60 = 41.25$  lands. The numbers won't match exactly, but they'll be close enough.

### **Musings** – Can you cut a land on the draw?

For sure. Suppose that in a 60-card deck, you'd want to hit your 3rd land drop with 90% certainty and your 4th land drop with 75% certainty. On the play, you need 26 lands to achieve this. On the draw, you only need 23 lands. The extra draw step makes a big difference!

Sideboarding in and out lands is something that many of the top pros do constantly, and the numbers in the table indicate that this is a valid strategy. It's possible that cutting as many as 2 lands might even be correct in some cases. But I wouldn't go as far as actually going from 26 to 23 lands, despite what the numbers might suggest. After all, such a reduction would come at the cost of more frequent mulligans (as you can observe in the column with the expected opening hand size). Moreover, you're generally more mana hungry on the draw because you may have one more spell than your opponent. Sometimes, you have to look beyond the numbers.

In Limited, cutting a land when you're on the draw is fine for the same reasons. But more so in Limited than in Constructed, you should check if you still have enough colored mana sources for each of your main colors before doing this.

### **Musings** – How to balance mana screw and mana flood?

Ultimately, the decision on how many lands to put in your deck is a trade-off between mana screws and mana floods. Calculating the probability of hitting 4 land drops by turn 4 is relatively easy—anyone with a basic knowledge of probability theory should be able to replicate my results with a simple spreadsheet or program—but adequately weighing the relative impact of mana screws and floods is more difficult. It also depends on your deck and the format.

My subjective judgment of saying that 83.5% is a good number for hitting your first 4 land drops on the draw is mostly based on experience and intuition, and it only applies to a certain type of deck. Unless you have tracked the results of thousands of games in a certain matchup, there is no scientific reason why 83.5% is “better” than 80% or 87%. It's just a number that looks good to me.

Besides, there are plenty of factors that should influence your land counts as well, such as whether or not your deck contains additional sources of mana (e.g., [Servant of the Conduit](#)) or ways to spend excess mana (e.g., [Duskwatch Recruiter](#)).

But no matter how many lands you play, mana screws and mana floods are part of the game. In my opinion, as long as they're relatively infrequent, they're actually good for the game. Little bits of variance give weaker players a chance to beat better players, lead to games that play out differently every time, add excitement to draw steps, and make for interesting deck building decisions.

But while a little bit of randomness is fun, too much randomness is not. If you lose too often to mana screw or mana flood, then that may not be enjoyable—you don't get to leverage your playing skill. Likewise, if random [Aetherworks Marvel](#) spins decide too many games, then players may feel that the game is out of their hands.

So it's about finding the right balance. Fortunately, R&D regularly offers nice aids to our mana bases.

Besides cycling lands, creaturelands, or land-search effects with an alternative ability in the late game, there are plenty of other cards that help mitigate mana flood, and these are invaluable tools for any Constructed deck. When you've drawn too many lands, these cards can still provide useful effects, allowing you to stay in the game even when you're flooded.

It is somewhat surprising to me that in comparison, there seem to be fewer cards that mitigate mana screw. To some extent, the above-mentioned cards help a little bit because they incentivize people to add more lands to their deck. Cycling lands, for instance, lead to slightly inflated land counts, and I think that's great—these cards should always be in Standard. But mana screws still happen from time to time, even with inflated land counts, and then there are not many cards that can help you.

I would love to see a card like this, for instance.



Printing a card like this for Standard may cause the mana screw probability (defined in a suitable way) to go down from, say, 20% to 10%. I'm pretty sure it shouldn't be 0%, as in *Hearthstone*, but cards like this Mountain Boar might help us get toward the sweet spot for maximum player satisfaction. Just a random thought.

## Method 2: Data Set

Back to the math. For my second method of providing insight into the question of how many lands you need, I went over the last 3 Pro Tours (*Kaladesh*, *Aether Revolt*, and *Amonkhet*). I grabbed all Top 8 deck lists and all deck lists outside of the Top 8 that went 8-2 or better in Standard. This yielded 78 deck lists in total.

For each of these decks, I noted down the number of lands (including, for this purpose, [Attune with Aether](#) and [Traverse the Ulvenwald](#)) and the average converted mana cost of nonland cards. Because nearly every one of these Standard decks had between 22 and 26 lands—with 24 the most common number by far, I then also added the Top 8 decks of the last 4 Modern Grand Prix events (Kobe, Copenhagen, Vancouver, and Brisbane, for a total of 32 decks) to get some decks with more extreme numbers of lands.

## Special Cases

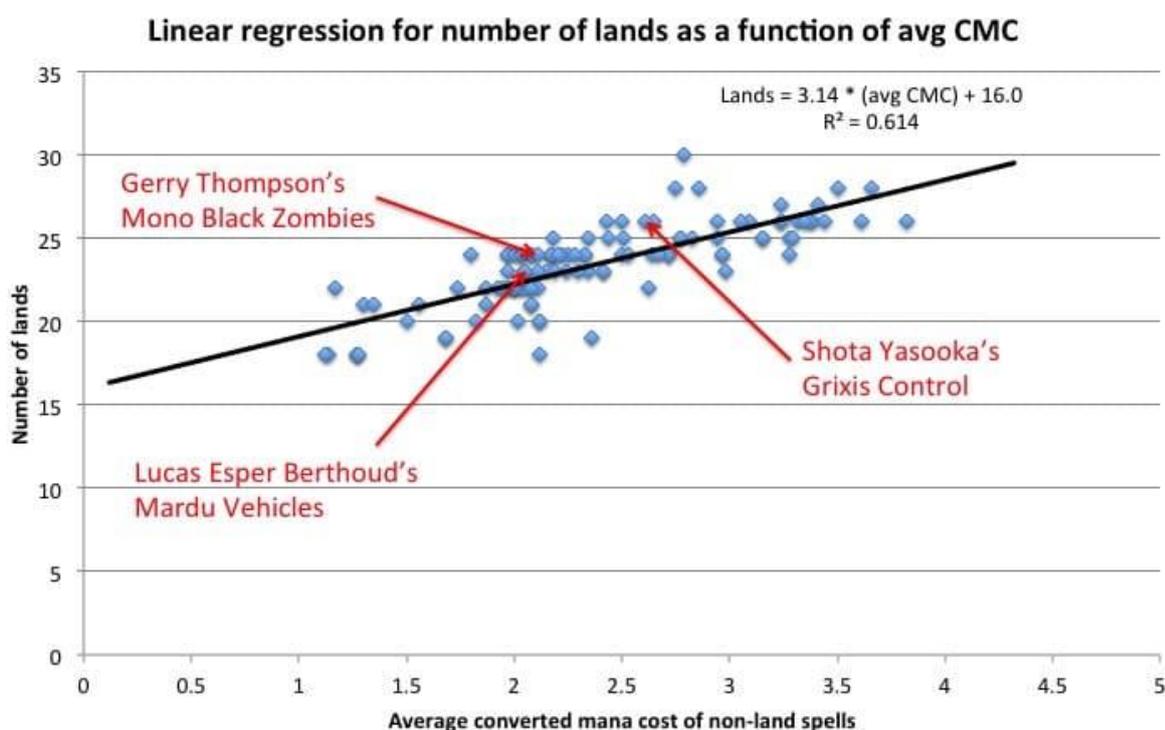
I treated special cases as follows:

- In Standard, cheap spells whose main aim is to fetch a land ([Attune with Aether](#), [Traverse the Ulvenwald](#), etc.) were counted as both a land and a 1-mana card. In Modern, I had a similar approach but I made an exception for [Traverse the Ulvenwald](#). Since delirium is so easy to achieve and creatures are so good, I did not count it as a land.
- 0-mana cards that produce mana (i.e., [Mox Opal](#)) were counted as a land. [Aether Vial](#) was also treated as a land.
- Mana creatures ([Servant of the Conduit](#), [Noble Hierarch](#), etc.) were not counted as lands. Mana rocks ([Mind Stone](#), [Cultivator's Caravan](#), etc.) or cheap card draw/selection spells ([Vessel of Nascency](#), [Serum Visions](#), [Mishra's Bauble](#), etc.) were also not considered a land.
- Cards with a reduced cost ([Bone Picker](#), [Tasigur](#), [Distended Mindbender](#), etc.) were counted for their full converted mana cost. The exception was [Emrakul, the Promised End](#), which I counted as an 8-mana card.
- For X-spells, I counted [Chalice of the Void](#) as a 0-mana card, [Dark Salvation](#) as a 1-mana card, [Walking Ballista](#) as a 2-mana card, and [Quarantine Field](#) as a 4-mana card.
- [Street Wraith](#) was counted as having a converted mana cost of 0. In the same vein, [Desert Cerodon](#), [Monstrous Carabid](#), and [Horror of the Broken Lands](#) were counted as 1-mana cards.

As you can see, a lot of simplifications and modeling choices were made and in reality, the truth is somewhere in the middle: [Attune with Aether](#) or [Aether Vial](#) are not full lands, and cards like [Noble Hierarch](#), [Cultivator's Caravan](#), or [Serum Visions](#) could easily count as half a land or one-third of a land. Likewise, [Peter Vieren's 24-land control deck](#) is closer to a 27-land deck thanks to eight 1-mana cyclers, but it was still seen as a 24-land deck under my assumptions. My method also counted [Ulamog, the Ceaseless Hunger](#) as a card with converted mana cost 10, which is not completely fair either. But with the time I had available, I was able to assemble a data set whose quality I would call sufficient.

## Linear Regression Results

On a scatterplot for all 110 decks, I plotted the best-fitting line for all the data points. Specifically, I used the least squares approach from simple linear regression to minimize the sum of the squared distances between the fitted line and all the data points. I have no reason to believe that the actual relationship has to be linear per se, but I also didn't have any reason to suspect a more complicated relationship. I got a reasonable R-squared value (loosely speaking, a statistical measure of how close the data points are to the fitted line) of 0.614, and the results are shown in the picture below.



The fitted model, based on my data set, is that the number of lands in a deck is given by 16 plus 3.14 times the average converted mana cost of its nonland spells. As you can see in the data sets, many decks have maybe 2 lands more or 2 lands fewer than this formula would indicate—in the end, every deck is different, cheap card drawing spells or mana creatures can influence the counts, and the average converted mana cost doesn't say everything. But you can at least get a rough guideline from this analysis.

An interesting observation is that the decks that won the last 3 Pro Tours, pointed out in red, all lie all above the regression line. Although I surely don't have a large enough sample size to make sweeping conclusions, it does mean that these Pro Tour winners did not skimp on their land counts—rather, they had slightly inflated land counts. Take that as an indication when you're in doubt, you're probably better off adding that extra land instead of that extra spell to your deck.

# Conclusion

Combining key numbers from both methodologies in a single table, I can offer a suite of recommendations for 60-card decks. For Commander, you can multiply the number of recommended lands by 99/60. For Limited, you can multiply by 40/60 to get close to a good number.

Note that to properly interpret the first column in this table, “number of lands,” you should count cards like [Attune with Aether](#) or Aether Vial. For more, see the assumptions described in the section “Method 2: Special Cases.”

Number of lands	Average CMC range	Type of deck (percentages are for being on the play)
18	0.48-0.80	<b>Low-curve deck</b> – You only need 1 or 2 lands to function, and you have no 3-drops in your deck
19	0.80-1.12	<b>Low-curve deck</b> – You need 2 lands on turn 2 (97.7%) but would like 3 lands on turn 3 (76.5%) for some 3-drops
20	1.12-1.44	<b>Low-curve deck</b> – You need 2 lands on turn 2 (98.3%) but would like 3 lands on turn 3 (79.6%) for some 3-drops
21	1.44-1.76	<b>Aggro deck</b> – You need 2 lands on turn 2 every game (98.8%) but would like 3 lands on turn 3 (82.3%) for several 3-drops
22	1.76-2.08	<b>Aggro deck</b> – You need 2 lands on turn 2 every game (99.2%) but would like 3 lands on turn 3 (84.7%) for several 3-drops
23	2.08-2.40	<b>Aggro deck</b> – You need 3 lands on turn 3 in most games (86.8%); 4 lands by turn 4 (67.7%) is nice but not necessary
24	2.40-2.72	<b>Midrange deck</b> – You need 3 lands on turn 3 in most games (88.7%); 4 lands by turn 4 is nice (71.3%) but not necessary
25	2.72-3.04	<b>Midrange deck</b> – You need your 3rd land drop (90.4%) to function and want to hit your 4 <sup>th</sup> relatively often (74.7%)
26	3.04-3.36	<b>Control deck</b> – You need your 3 <sup>rd</sup> land drops (91.8%) to function and want to hit your 4 <sup>th</sup> relatively often (77.9%)
27	3.36-3.68	<b>Control deck</b> – You need your 4 <sup>th</sup> land drop (80.8%) to function and regularly want to hit your 5 <sup>th</sup> (65.3%)

I hope that my analysis will prove useful to deck builders all around the world. Magic is more fun when you have the right number of lands in your deck—don't make the mistake of skimping on your land counts!

### **About The Author**

*Frank Karsten has been playing competitive Magic for over half his life, and he was inducted into the Pro Tour Hall of Fame in 2009. Synergy-driven aggro decks like Modern Affinity are his favorite. Frank holds a PhD in cooperative game theory and stochastic operations research, so when he's not traveling the world, he enjoys applying his mathematical background to Magic-related problems.*